Coordinate Geometry

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Coordinate Geometry
Booklet	Question Paper

Time Allowed: **88 minutes**

Score: **/73**

Percentage: **/100**

Grade Boundaries:

1 A circle has equation $(x-4)^2+(y+7)^2=64$.

(i) Write down the coordinates of the centre and the radius of the circle. [2]

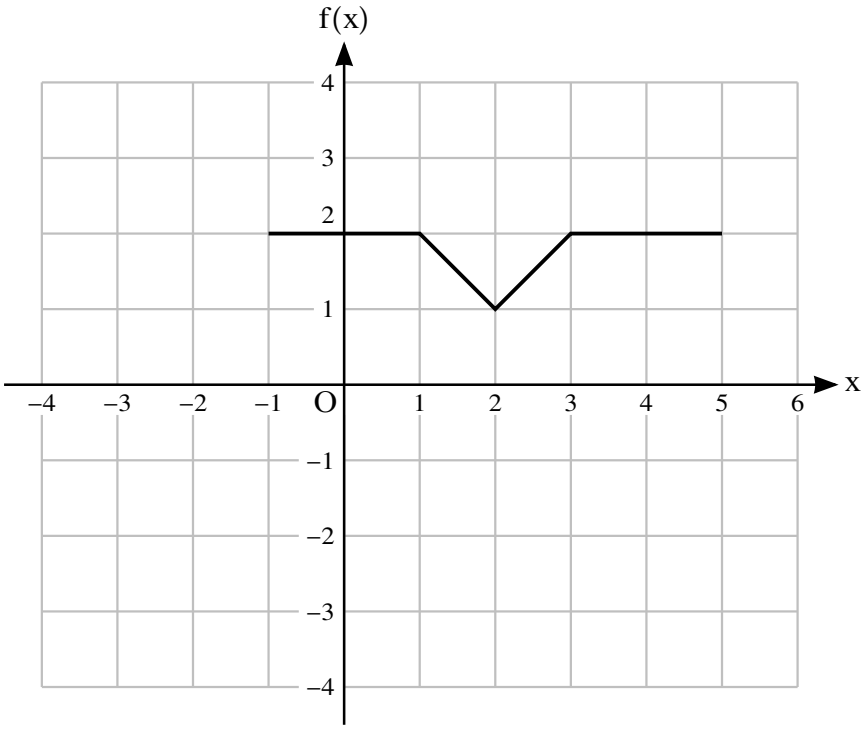
Two points, A and B, lie on the circle and have coordinates (4, 1) and (12, –7) respectively.

(ii) Find the coordinates of the midpoint of the chord AB. [2

2 **(i)** The points A and B have coordinates (–4, 4) and (8, 1) respectively. Find the equation of the line AB. Give your answer in the form $y = mx + c$. [3]

(ii) Determine, with a reason, whether the line $y = 7 – 4x$ is perpendicular to the line AB. [2

3 The graph of f(x) is shown below.



Draw the graphs of

(i) f(x + 2) + 1, [2]

(ii) $-\frac{1}{2}$f(x). [2]

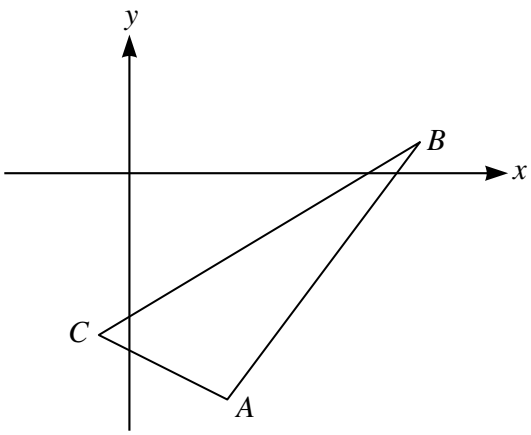
4 The points A, B, C and D have coordinates (2, −1, 0), (3, 2, 5), (4, 2, 3) and (−1, a, b) respectively, where a and b are constants.

(i) Find the angle ABC. [4]

(ii) Given that the lines AB and CD are parallel, find the values of a and b. [3]

5 A is the point (2, 1) and B is the point (10, 7). Find the coordinates of the mid-point of AB and the length of AB. [3]

6



The diagram shows a triangle ABC. The vertices have coordinates A (3, −7), B (9, 1) and C (−1, −5).

(i) **(a)** Find the length of the side AB. [2]

(b) Find the coordinates of the mid-point of AB. [1]

(c) A circle has diameter AB. Find the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$, where a, b and r are constants to be found. [3]

(ii) Find the equation of the line l passing through B parallel to AC. [3]

7 Find the equation of the line passing through the points $(-2, 5)$ and $(4, -7)$. Give your answer in the form $y = mx + c$. [3]

8 Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\begin{aligned} \text{f} &: x \mapsto x^2 - 2x, \\ \text{g} &: x \mapsto x^2, \\ \text{h} &: x \mapsto \sin x. \end{aligned}$$

(i) (a) State whether or not f has an inverse, giving a reason. [2]

(b) Determine the range of the function f. [2]

(ii) (a) Show that $\text{gh}(x)$ can be expressed as $\frac{1}{2}(1 - \cos 2x)$. [2]

(b) Sketch the curve C defined by $y = \text{gh}(x)$ for $0 \leqslant x \leqslant 2\pi$. [3]

9 The curve $y = x^2$ intersects the line $y = kx$, $k > 0$, at the origin and the point P. The region bounded by the curve and the line, between the origin and P, is denoted by R.

(i) Show that the area of the region R is $\frac{1}{6}k^3$. [3]

The line $x = a$ cuts the region R into two parts of equal area.

(ii) Show that $k^3 - 6a^2k + 4a^3 = 0$. [3]

The gradient of the line $y = kx$ increases at a constant rate with respect to time t. Given that $\frac{\mathrm{d}k}{\mathrm{d}t} = 2$,

(iii) determine the value of $\frac{\mathrm{d}a}{\mathrm{d}t}$ when $a = 1$ and $k = 2$, [4]

(iv) determine the value of $\frac{\mathrm{d}a}{\mathrm{d}t}$ when $a = 1$ and $k \neq 2$, expressing your answer in the form $p + q\sqrt{3}$, where p and q are integers. [5]

10 The point F has coordinates $(0, a)$ and the straight line D has equation $y = b$, where a and b are constants with $a > b$. The curve C consists of points equidistant from F and D.

(i) Show that the cartesian equation of C can be expressed in the form

$$y = \frac{1}{2(a-b)}x^2 + \tfrac{1}{2}(a+b).$$ [3]

(ii) State the y-coordinate of the lowest point of the curve and prove that F and D are on opposite sides of C. [2]

(iii) **(a)** The point P on the curve has x-coordinate $\sqrt{a^2 - b^2}$, where $|a| > |b|$. Show that the tangent at P passes through the origin. [4]

(b) The tangent at P intersects the line D at the point Q. In the case that $a = 12$ and $b = -8$, find the coordinates of P and Q. Show that the length of PQ can be expressed as $p\sqrt{q}$, where $p = 2q$. [5]